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The Passage of a Detonation Wave across the Interface between Two Explosives

H. H. M. Pike and R. E. Weir

Fort Halstead,
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The Passage of a Detonation Wave across the Interface between Two Explosives

H.H.M. Pike and R.E. Weir

Summary

This report is divided into two parts. In the first we consider what happens when a plane detonation wave crosses a plane interface, parallel to that front, between two different explosives. The case of most interest is that in which the first explosive overdrives the second, the pressure and detonation velocity in the second explosive having values higher than normal and the Chapman-Jouguet condition not being satisfied. This is known as the "carry-over" effect and it would persist indefinitely if the two explosives were infinite in extent, but for charges of finite size it fades away in a distance comparable with charge dimensions. The effect of the finite reaction rate is to produce a zone, of thickness a few millimetres, which the detonation front must traverse before the carry-over effect is fully established.

Criteria are given for deciding whether or no carry-over will occur in any particular case, and numerical results are given for a few cases in which it does occur. The most interesting result is that carry-over produces a far greater increase in pressure than in detonation velocity.

The second part deals with oblique incidence of the detonation front on the interface. Only steady cases are considered and no essentially new result is obtained. The magnitude of the carry-over effect varies with angle of incidence and disappears for sufficiently large departures from head-on incidence.

Two subsidiary results of some interest are derived. The first is that the Chapman-Jouguet condition is not sufficient to ensure a steady detonation velocity if the dominant reaction near the end of the reaction zone is of zero order. If the reaction is of higher order then it is never quite completed and we have to define reaction-zone thickness in a manner analogous to the definition of shock-front thickness. The second is that the intersection of two detonation waves inside an explosive is always regular, no Mach wave being produced.

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Mass $\rho(w-d) = -\rho_0 d$ $\quad w = d\left(1 - \frac{\rho_0}{\rho}\right)$

Mom $p + \rho(w-d)^2 = p_0 + \rho_0 d^2$ $\quad w^2 = d^2\rho_0^2\left(\frac{1}{\rho_0} - \frac{1}{\rho}\right)^2$

$$p - p_0 = \rho_0 d^2 + \rho_0 d(w-d)$$

$$= \rho_0 w d$$

$$= \rho_0 d^2\left(1 - \frac{\rho_0}{\rho}\right) = \rho_0^2 d^2\left(\frac{1}{\rho_0} - \frac{1}{\rho}\right)$$

$$\therefore\ d^2 = V_0^2 \frac{(p-p_0)}{V_0 - V} \qquad (2)$$

$$\therefore\ w^2 = (p-p_0)(V_0 - V) \qquad (1)$$

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_S = -\frac{1}{\rho^2}\left(\frac{\partial p}{\partial V}\right)_S = -V^2\left(\frac{\partial p}{\partial V}\right)_S$$

$$\frac{c^2}{V^2} = -\left(\frac{\partial p}{\partial V}\right)_S = -\frac{\partial(p,S)}{\partial(V,S)} = -\frac{\partial(p,S)}{\partial(V,E)}\frac{\partial(V,E)}{\partial(V,S)} = -\frac{\partial(p,S)}{\partial(V,E)}\left(\frac{\partial E}{\partial S}\right)_V = -T\frac{\partial(p,S)}{\partial(V,E)}$$

$$= -T\begin{bmatrix}\left(\frac{\partial p}{\partial V}\right)_E & \left(\frac{\partial p}{\partial E}\right)_V \\ \left(\frac{\partial S}{\partial V}\right)_E & \left(\frac{\partial S}{\partial E}\right)_V\end{bmatrix} = -T\begin{bmatrix}\left(\frac{\partial p}{\partial V}\right)_E & \left(\frac{\partial p}{\partial E}\right)_V \\ \frac{p}{T} & \frac{1}{T}\end{bmatrix} = -\begin{bmatrix}\left(\frac{\partial p}{\partial V}\right)_E & \left(\frac{\partial p}{\partial E}\right)_V \\ p & 1\end{bmatrix} = -\left(\frac{\partial p}{\partial V}\right)_E + p\left(\frac{\partial p}{\partial E}\right)_V$$

(5)

Part I. The Head-On Case

1. Introduction

It has been observed experimentally that when a detonation wave passes from one explosive to another its velocity in the second explosive may not settle down to its usual value until the wave has travelled a distance of several centimetres. This "carry-over" effect, as it is called, does not always occur and it is the purpose of the first part of this report to explain the phenomenon and to give a guide for deciding whether or not it will occur.

We treat detonation waves in the first place as perfectly one-dimensional i.e. as plane waves travelling in explosives of infinite extent, the plane of the detonation front being parallel to the interface between the two explosives. Modifications due to the finite dimensions of actual explosives are then dealt with quite briefly. In a detonation wave the transition from undisturbed conditions ahead of the wave to steady conditions behind it occurs in the reaction zone, whose thickness is of the order of a millimetre. When the front of this zone strikes the interface a shock or rarefaction wave may pass back through the zone, so disturbing the reaction. There will therefore be a narrow zone of the first explosive, of thickness less than a millimetre, whose properties may be different from those of the bulk of the products of detonation of that explosive. The same may also be true of a narrow zone of the second explosive adjoining the interface. In the extreme case when the second explosive is replaced by a vacuum part of this narrow zone in the first explosive may not react at all. We shall ignore this intermediate zone in the first place i.e. treat the reaction zone in either explosive as being infinitely thin, a brief discussion will then show that in the cases which we have in mind the presence of the intermediate zone may delay the setting-up of the proper carry-over conditions so that the full effect may not be quite attained in charges of finite dimensions.

We shall first assume that there is no carry-over and determine the conditions to ensure this. When these conditions are not satisfied we shall show how to calculate the magnitude of the carry-over effect and illustrate the method by a series of examples.

In the second part of this report the same methods are applied to cases where the detonation front is not parallel to the interface. Only steady cases are considered and no essentially new phenomenon is introduced. Some examples are givenfor a particular pair of explosives. For some angles of incidence depending on the explosives used, steady conditions are not possible but we may get a quasi-steady solution starting from a point on the interface and with all linear dimensions growing at a constant rate.

2. The essential equations

We start with explosive at rest at atmospheric pressure p_0 and specific volume v_0, through which a shock front travels with velocity d. The front is followed by a very thin reaction zone in which an amount of chemical energy q' is liberated per unit mass of material passing through. Since the flow is one-dimensional, conservation of mass, momentum and energy yield the three relations

w = particle velocity or material velocity

$$w^2 = (p - p_0)(v_0 - v) \tag{1}$$

$$d^2 = v_0^2(p - p_0)/(v_0 - v) \tag{2}$$

and

$$e - e_0 = q' + \tfrac{1}{2}(p + p_0)(v_0 - v) \tag{3}$$

where e is the internal energy per unit mass.

For a pure shock $q' = 0$. We assume q' to be given in any case and e to be a known function of p and v. For a given v_0 we then have three equations for the four unknowns w, d, v and p, so that given any one of these the others are determined. For a pure detonation wave we have the Chapman-Jouguet condition

$$d = w + c \tag{4}$$

where c is the local velocity of sound. We can express c in terms of p and v,

$$\frac{c^2}{v^2} = -\left(\frac{\partial p}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial v}\right)_e + p\left(\frac{\partial p}{\partial e}\right)_v \tag{5}$$

which is known since e is expressible in terms of p and v. This extra condition is sufficient to determine all the unknowns.

For gases $e = pv/(\gamma - 1)$ to a good approximation and γ is of order 1.3. For solid explosives a fair approximation can be had by putting $e = pv/2$ i.e. $\gamma = 3$. For gases p_0v_0/d^2 is usually of order .01 to .02 while for solid explosives it is of order 10^{-5}. If we neglect terms of this order in comparison with terms of order unity then the solutions of the equations can be put in the simple approximate form

$$p_2 \simeq 2(\gamma - 1)\, q/v_0 \tag{6}$$

$$v_2 \simeq \gamma\, v_0/(\gamma + 1) \tag{7}$$

$$d_2^2 \simeq 2(\gamma^2 - 1)\, q \tag{8}$$

and

$$w_2 = d_2/(\gamma + 1)$$

or

$$w_2^2 \simeq 2(\gamma - 1)\, q/(\gamma + 1) \tag{9}$$

For convenience we have added e_0 to q', i.e.

$$q = q' + e_0 \tag{10}$$

e_0 is usually of order 5 to 10% of q and so should not be neglected. The approximation $e_0 = p_0v_0/(\gamma - 1)$ is fairly good for gases but too small by a factor < .01 for solid explosives.

These equations only suffice for a very qualitative discussion. We can however, use equally simple relations with reasonable accuracy if we choose γ to give the correct isentropic variation of p with v for conditions near to those obtaining in the Chapman-Jouguet plane under normal detonation conditions.

We use suffix 2 to indicate exact solutions of the detonation equations (1) to (5). Let suffix 3 indicate some other solution of (1) to (3) but not (4), i.e. an unstable detonation wave. We shall only consider cases where $p_3 > p_2$ i.e. $d_3 > d_2$. Then substituting both solutions in (3) and subtracting we have

$$e_3 - e_2 = \tfrac{1}{2} p_3(v_0 - v_3) - \tfrac{1}{2} p_2(v_0 - v_2) + \tfrac{1}{2} p_0(v_2 - v_3) \tag{11}$$

For solid explosives the p_0 term is quite negligible. Provided we know p_2 and v_2 this gives a relation between p_3 and v_3 so that given the former we can determine w_3 and d_3 from (1) and (2). We shall be interested mainly in values of p_3 less than twice p_2 and we can show that if $(p_3 - p_2)/p_2$ is a small quantity then the difference in entropy between the two states is a small quantity of a higher order. We are therefore justified in writing

$$e_3 - e_2 = (p_3 v_3 - p_2 v_2)/(\gamma - 1) \tag{12}$$

where $$\gamma = c_2^2/p_2 v_2 = c_2/w_2 \tag{13}$$

and (12) holds with good accuracy for any explosive provided p_3 is not much greater than p_2. Substituting (12) in (11) we have

$$\frac{\gamma + 1}{\gamma - 1}(p_3 v_3 - p_2 v_2) = (p_3 - p_2)v_0 + p_0(v_2 - v_3) \tag{14}$$

where the p_0 term may be dropped for dense explosives. In that case we may write

$$w_3^2 = v_0(2\,p_3 - p_2)/(\gamma + 1)$$

and $$d_3 = p_3 v_0/w_3 = d_2 \Big/ \sqrt{\left\{\frac{p_2}{p_3}\left(2 - \frac{p_2}{p_3}\right)\right\}} \tag{15}$$

If a pure shock travels through the detonation products, producing a peak pressure p_3 then v_3 and w_3 are given by equations (1) to (3) with suffix 0 replaced by suffix 2. The velocity of the products in condition 2 has now to be added to w_3. If p_3 is not much greater than p_2 we may again use γ as defined by (13); this leads to

$$|w_3 - w_2| = (p_3 - p_2)\sqrt{[2\,v_2/\{(\gamma + 1)p_3 + (\gamma - 1)p_2\}]} \tag{16}$$

On the other hand a pure rarefaction wave produces a change in velocity

$$|w_3 - w_2| = \frac{2\,c_2}{\gamma - 1}\left\{1 - \left(\frac{p_3}{p_2}\right)^{(\gamma-1)/2\gamma}\right\} \tag{17}$$

3. The conditions for no carry-over

We assume first of all that the reaction zone is of negligible thickness and that the detonation front has passed the interface between two explosives, changing instantly to conditions appropriate to a steady plane detonation wave in the second explosive. Let us use large letters to denote conditions in the products of detonation of the first explosive and small ones for the second. In general $P_2 \neq p_2$, $W_2 \neq w_2$ and so there must be a wave, spreading out in both directions from the interface, of such a nature as to make pressure and fluid velocity continuous across the interface. Four possible cases can arise since each edge of the wave may be either a shock front or the leading edge of a rarefaction wave. If the wave going forward into the products of the second explosive is a rarefaction wave its leading edge will travel with velocity c_2 and, owing to the Chapman-Jouguet condition, it will never be able to pass through the reaction zone but will follow immediately behind it. The velocity of detonation will therefore be unaffected but the pressure behind the wave front will fall rapidly to some value less than p_2. On the other hand if the leading edge of the wave is a shock this will travel at supersonic speed and so will penetrate through the Chapman-Jouguet plane and increase the detonation velocity. There will then be a region of uniform pressure $p_3 > p_2$ behind the front and extending back to the interface.

The definition of carry-over is a change in detonation velocity; the necessary condition for no carry-over is therefore that the pressure at the interface, $p_3 < p_2$. Whether the edge of the wave going back into the second explosive is a shock or a rarefaction is only of importance in so far as it affects the conditions governing the nature of the forward-moving edge.

Since there are four cases the existence of carry-over in any particular case does not imply its absence when the explosives are interchanged. The converse is also true.

For brevity we write π for p_3/p_2 and Π for p_3/P_2 (since $p_3 = P_3$). If there is no carry-over we have, since the rarefaction reduces w

$$w_3 = w_2 - \frac{2\,c_2}{\gamma - 1}\left\{1 - \pi^{(\gamma-1)/2\gamma}\right\} \tag{18}$$

and if the backward moving wave is a shock then

$$W_3 = W_2 - (\Pi - 1)\sqrt{[2\,P_2V_2/\{(\Gamma + 1)\Pi + \Gamma - 1\}]} \tag{19}$$

The following argument is taken from Paterson (1948):- w_3 is a monotonic increasing function of π, being equal to w_2 when $\pi = 1$. Again W_3 is a monotonic decreasing function of Π, and Π increases or decreases in proportion to π when p_2 and P_2 are fixed. Hence the condition that $\pi < 1$ when $W_3 = w_3$ is equivalent to $w_3 > W_3$ for $\pi = 1$, i.e.

$$w_2 - W_2 > -(p_2 - P_2)\sqrt{[2\,V_2/\{(\Gamma + 1)p_2 + (\Gamma - 1)P_2]} \tag{20}$$

This is the governing condition only when $\Pi > 1$, which is equivalent to $W_3 > w_3$ for $\Pi = 1$, i.e.

$$w_2 - W_2 < \frac{2\,c_2}{\gamma - 1}\left\{1 - \left[\frac{P_2}{p_2}\right]^{(\gamma-1)/2\gamma}\right\} \tag{21}$$

If (21) is not fulfilled then a rarefaction wave accelerates the first fluid forward i.e. $W_3 > W_2$ and (20) must be replaced by

$$w_2 - W_2 > \frac{2\,C_2}{\Gamma - 1}\left\{1 - \left[\frac{p_2}{P_2}\right]^{(\Gamma-1)/2\Gamma}\right\} \tag{22}$$

When the equality in the primary condition (21) is satisfied conditions (20) and (22) are equivalent although not quite identical.

If we use the approximate solution (6) to (10) of the detonation equations then (21), (20) and (22) become either

$$-\frac{2\,V_0}{\Gamma+1}\left\{\frac{(\gamma-1)q}{v_0} - \frac{(\Gamma-1)Q}{V_0}\right\}\Big/\sqrt{\left\{\frac{\Gamma\,v_0}{2(\gamma-1)q\,V_0 + 2(\Gamma-1)^2Qv_0/(\Gamma+1)}\right\}} < \sqrt{\left\{\frac{\gamma-1}{\gamma+1}\,q\right\}} - \sqrt{\left\{\frac{\Gamma-1}{\Gamma+1}\,Q\right\}} < 2\gamma\sqrt{\left\{\frac{q}{\gamma^2-1}\right\}}\left\{1 - \left[\frac{(\Gamma-1)Q\,v_0}{(\gamma-1)q\,V_0}\right]^{(\gamma-1)/2\gamma}\right\}$$

or alternatively if

$$\sqrt{\left\{\frac{\gamma-1}{\gamma+1}\,q\right\}} - \sqrt{\left\{\frac{\Gamma-1}{\Gamma+1}\,Q\right\}} > 2\gamma\sqrt{\left\{\frac{q}{\gamma^2-1}\right\}}\left\{1 - \left[\frac{(\Gamma-1)Q\,v_0}{(\gamma-1)q\,V_0}\right]^{(\gamma-1)/2\gamma}\right\}$$

then it must also be greater than or equal to the same expression with large and small letters interchanged.

We have reduced the condition for no carry-over to a condition on the three independent parameters specifying an explosive namely q, the heat of detonation per unit mass, v_0 the initial specific volume and γ the adiabatic exponent for the products. For gas mixtures γ will be of order 1.3 but will vary somewhat. We can get a very rough guide to the behaviour of solid explosives if we take $\Gamma = \gamma = 3$ and then the conditions simplify to either

$$(Q\,v_0 - q\,V_0)/\sqrt{\{\tfrac{1}{3}\,v_0(2q\,V_0 + Q\,v_0)\}} < \sqrt{(q)} - \sqrt{(Q)} < 3\sqrt{(q)}\cdot\{1 - (Q\,v_0/q\,V_0)^{1/3}\}$$

or alternatively if

$$\sqrt{(q)} - \sqrt{(Q)} > 3\sqrt{(q)}\{1 - (Q\,v_0/q\,V_0)^{1/3}\}$$

then it must also be greater than or equal to the same expression with large and small letters interchanged.

For two explosives with the same initial density we get no carry-over if $q > Q$ and vice versa. Again if $q = Q$ we get no carry-over if the initial density of the second explosive exceeds that of the first. These results are almost obvious.

We have determined the necessary conditions for no carry-over. If we consider all the cases that can arise we can show that they are sufficient.

4. Calculation of the initial detonation velocity when carry-over occurs

We have determined the conditions necessary to prevent carry-over and we have also seen that when carry-over does occur the Chapman-Jouguet condition is no longer satisfied. We then get an increased detonation velocity, which would be permanent in the ideal plane-wave case. In practical cases however, the first explosive is of limited length and in consequence there is a pressure gradient behind the detonation wave as it travels through the first explosive, the pressure falling steadily with distance behind the reaction zone. Owing to the Chapman-Jouguet condition this rarefaction wave cannot affect the detonation velocity in the first explosive but if the Chapman-Jouguet condition is not observed in the early stages in the second explosive this rarefaction wave will pass through the reaction zone and reduce the detonation velocity until Chapman-Jouguet conditions are restored. If the explosive is of finite cross-section then rarefaction waves coming in from the sides increase the pressure gradient behind the front. The distance the detonation wave travels in the second explosive before the carry-over entirely disappears will therefore be comparable with the least dimension of either explosive.

5. The effect of finite reaction rate

In connection with studies of initiation Eyring et.al. (1949) have calculated the rate at which a detonation wave, started at the wrong velocity, will build-up, or build-down, to the steady value. Their calculations are based on the assumption that at any instant d equals the sum, $c + w$, obtaining at the rear of the reaction zone a short time τ earlier. This leads immediately to d approaching its correct value in a time of order 3τ. τ is defined as approximately the time taken for a fluid element to pass through the reaction zone, but to this should be added the time taken for a signal to pass from the rear of the reaction zone to the front of the wave, which may be considerably longer since it has to travel against the stream. It is clear that the settling-down distance in the infinite-plane case must increase roughly in proportion to τ but difficult to deduce the factor of proportionality; Eyring's theory is too crude to do more than indicate the order of magnitude.

The following discussion may help to clarify our ideas. The detonation wave is led by a strong shock front, which is so very thin that inside it no appreciable amount of reaction occurs. The velocity of this shock front and so of the detonation wave as a whole depends only on the shock pressure and the mechanical properties of the unreacted explosive, but the energy necessary to prevent that shock from decaying comes from the reaction. Its velocity does in fact depend on conditions at every point of the reaction zone. Different parts of the reaction zone also depend on each other. If therefore, the system is in equilibrium until at some time a disturbance originates at some point in the reaction zone, then waves will travel out from that point in both directions with a velocity nearly equal to the local velocity of sound superimposed on the local material velocity. The backward-travelling wave will reach the back of the reaction zone very quickly and change conditions there. A new wave must then move forward through the entire length of the reaction zone to change the velocity of the shock front. This forward moving wave travels at a speed very little different from that of the front itself, in its early stages and so will take a comparatively long time τ to reach the front. The detonation velocity will therefore settle down to its new value in a time comparable with τ, say 3τ.

The wave will spend most of the time τ travelling through the first 10% or so of the reaction zone length l. We can therefore form an estimate of τ if we simplify the problem by using approximations valid for the rear part of the reaction zone. For our present purposes it will suffice to use the ideal-gas equation of state; we can then make use of the results obtained for the steady flow of reacting gases through ducts by Chambre and Lin (1946). These authors made an error in the effect of variation of cross-section of the duct, which is given correctly by Hicks et.al. (1947).

During the reaction the number of moles n of gas per unit mass will change and this change may be just as important as the release of thermal energy. For steady plane parallel flow we have

$$\begin{aligned}
\frac{du}{u} &= \frac{1}{\gamma(1 - M^2)}\left\{\frac{dq}{e} + \frac{dn}{n}\right\} \\
\frac{dc}{c} &= \frac{1 - \gamma M^2}{2\gamma(1 - M^2)}\left\{\frac{dq}{e} + \frac{dn}{n}\right\} \qquad (23) \\
\frac{dM^2}{M^2} &= \frac{1 + \gamma M^2}{\gamma(1 - M^2)}\left\{\frac{dq}{e} + \frac{dn}{n}\right\}
\end{aligned}$$

where M is the local Mach number u/c. Making the approximations $u = u_2$, $c = c_2$, $n = n_2$ and $M = 1$ wherever possible we find since $u_2 = c_2$

$$1 - M^2 = \frac{2}{c_2}(c - u)$$

so that on substituting for $(1 - M^2)$ and integrating again we have to first order

$$\left\{\frac{c - u}{c_2}\right\}^2 = \frac{\gamma + 1}{2\gamma}\left\{\frac{q_2 - q}{e_2} + \frac{n_2 - n}{n_2}\right\}$$

Near the end of the reaction zone we may expect one type of reaction to predominate and so we write

$$\frac{n_2 - n}{n_2} = k\,\frac{q_2 - q}{e_2}$$

Using equations (1) to (10) we have

$$\begin{aligned}
c - u &= \frac{\gamma + 1}{2\gamma}\, c_2 \sqrt{\{(1 + k)(1 - q/q_2)\}} \\
&= \tfrac{1}{2}\, d_2 \sqrt{\{(1 + k)(1 - q/q_2)\}}
\end{aligned}$$

where q is here the heat liberated up to some definite point in the reaction.

If x is the distance from the rear of a reaction zone of length l then the time of passage of a very weak signal from the rear to the front of the wave

$$\tau = \int_0^l dx/(c - u)$$

If the final reaction is of zero order then to a sufficient approximation we may write

$$q/q_2 = 1 - b\, x/l$$

where b is a constant. On integrating we get

$$\tau = \frac{1}{d_2} \sqrt{\{b(1 + k)\}} \tag{24}$$

For any higher order depending on x we should find $\tau = \infty$ i.e. that no infinitesimal disturbance could get through the Chapman-Jouguet plane. The fact that τ is finite for a zero-order reaction means that in such a case the detonation wave is not truly stable.

Any higher-order reaction will never go to completion in a finite time i.e. the zone will be of infinite thickness. In such cases it is convenient to define a conventional reaction-zone thickness on the same lines as that proposed for shock-front thicknesses by Taylor and Maccoll (1935). If q' is the total heat liberated in the reaction then on dividing q' by the maximum value of dq'/dx we get a length l which we can define as the reaction-zone thickness. It is, of course, always an underestimate.

Since any higher order reaction is never completed we are forced to take into account the finite size of the charge. We then find that the detonation front is no longer plane, but slightly convex when viewed from the non-reacted side (Eyring, loc.cit.) and the curvature of the central portion determines the speed of propagation of the wave. The curved shock front causes the material flowing through it to diverge slightly and we may still treat the flow as one-dimensional but in a slowly diverging duct. If a is the cross-section of the duct then equations (23) now become

$$\begin{aligned}
\frac{du}{u} &= \frac{1}{\gamma(1 - M^2)} \left\{\frac{dq}{e} + \frac{dn}{n} - \gamma \frac{da}{a}\right\} \\
\frac{dc}{c} &= \frac{1 - \gamma M^2}{2\gamma(1 - M^2)} \left\{\frac{dq}{e} + \frac{dn}{n} - \gamma \frac{da}{a}\right\} + \frac{1}{2}\frac{da}{a} \\
\frac{dM^2}{M^2} &= \frac{1 + \gamma M^2}{\gamma(1 - M^2)} \left\{\frac{dq}{e} + \frac{dn}{n} - \gamma \frac{da}{a}\right\} - \frac{da}{a}
\end{aligned} \tag{25}$$

Since by definition

$$\frac{dM^2}{M^2} = 2\left\{\frac{du}{u} - \frac{dc}{c}\right\}$$

and $u_2 = c_2$ we have as before

$$1 - M^2 = 2(c - u)/c_2$$

while on integrating again we have approximately

$$\left\{\frac{c - u}{c_2}\right\}^2 = \frac{\gamma + 1}{2\gamma} \left\{\frac{q_2 - q}{e_2} + \frac{n_2 - n}{n_2} - \gamma \frac{a_2 - a}{a_2}\right\} - 2\int_a^{a_2} \frac{c - u}{c_2}\frac{da}{a_2} \tag{26}$$

Now the differential coefficient of the expression inside the bracket on the right hand side is zero on the Chapman-Jouguet surface. For sufficiently small values of x the right hand side can therefore be expressed as a polynomial in x, the lowest order term being A^2x^2 where A is a constant. A^2 includes a contribution from the integral. Hence to first order

$$(c - u)/c_2 = Ax$$

and we still find that an infinitesimal disturbance takes a logarithmically infinite time to get through the reaction zone.

Any finite disturbance Δ that can be produced by a combination of simultaneous changes in q, n and a (these can force a change in pressure), gives an initial value of $(c - u)$ proportional to $\sqrt{\Delta}$, which amounts to almost the same thing as starting with an initial value of x proportional to $\sqrt{\Delta}$. The transit time τ is now proportional to $\log \sqrt{(\Delta_1/\Delta)}$ i.e. to $-\log(\Delta/\Delta_1)$, where Δ_1 is a constant. We may include in τ the relatively short time taken by a signal to travel back from the shock front to the rear. It is then plausible to assume that in every interval τ the disturbance is reduced by a constant factor f, of order 2 or 3. Of course as the strength of the disturbance diminishes the time interval τ increases and the strength of the disturbance at any point diminishes in a series of jumps at successive time intervals. For the average rate of decrease we have

$$-\frac{d \log(\Delta/\Delta_1)}{dt} = \frac{\log f}{\tau} = \frac{-B}{\log(\Delta/\Delta_1)}$$

where B is a constant of proportionality. This integrates to give

$$\Delta = \Delta_1 \exp - \sqrt{\{2B(t - t_1)\}} \tag{27}$$

i.e. the disturbance diminishes exponentially with $\sqrt{t}$. We have found that a disturbance to the detonation velocity dies away much more rapidly than a disturbance to a thermodynamic property of the material, such as the pressure [c.f. equation (28)]. Presumably this could be allowed for by using a larger parameter B when Δ represents the disturbance to $d[\Delta d \sim (\Delta p)^2]$.

When a detonation wave passes across the interface between two explosives the initial conditions in the second explosive will usually be far from equilibrium, i.e. $(c - u)/d$ will be quite large (but < 1 of course) and so the bulk of the disequilibrium will disappear while the front travels a few reaction zone lengths (as defined above) and the detonation velocity will then approximate either to the normal value d_2 or, if carry-over obtains, to a value d_3 related to the pressure p_3 in the products of detonation. This pressure p_3 is itself falling however, owing to expansion of the detonation products, both radially and back along the axis of the charge. If the charge dimensions are all very large d_3 will fall in step with p_3 but if p_3 falls quickly d_3 will lag behind owing to the time delay through the reaction zone. As $d_3 \to d_2$ this time lag will increase and the late stages will always be governed by a law of the form (27) where $\Delta = d_3 - d_2$.

This time lag due to the finite reaction zone thickness implies that the full carry-over conditions as calculated in this paper will never be quite reached although they will be very nearly reached in large charges of fast reacting explosives.

6. Some numerical examples

Let us assume that for our two explosives we have d_2, p_2, v_2 and hence w_2, c_2 and γ, as calculated by some accurate equation of state, and that we wish to calculate the initial values p_3 and d_3 for the unsteady wave. Substitution in (21) and (20) or (22) will show us whether carry-over exists and if so whether the backward travelling wave is a shock or a rarefaction. Using either (19) or the corresponding rarefaction equation we can calculate W_3 as a numerical function of P_3 and graph our results. Then from (15) we can graph w_3 as a function of p_3. Where the two curves intersect gives the required values of p_3 and w_3, and therefore of d_3 from (2).

The Kistiakowsky-Wilson equation of state predicts detonation velocities for tetryl in good agreement with experimental results. We have therefore made use of the figures given for this explosive by Brinkley and Wilson (1943) to calculate the initial detonation velocity and pressure when tetryl at an initial density of 1.6 gm./c.c. is followed by the same explosive at a lower density. The results are shown in figures 1 and 2. In figures 3 and 4 similar results are given for PETN, based on calculations of Jones (1950) using a very general equation of state. In this case the first or "driving" explosive had an initial density of 1.727 gm./c.c.

The figures give the normal detonation velocity d_2 and pressure p_2 for each explosive when fired alone, for a series of values of initial density $1/v_0$, and also the enhanced values d_3 and p_3 when the explosive is preceded by the same material at its highest initial density. In both cases they show that the change in pressure is much more marked than the change in detonation velocity provided the difference in initial density is not too great.

If we regard v_0 as a variable then equations (1), (2) and (3), which apply to any one-dimensional wave, enable us at least in principle to express any three of the unknown quantities w, d, v and p in terms of the fourth together with v_0; in particular we could express d as a function of v_0 and p. Now the results of many hundreds of experiments prove that, for a given explosive fired alone under nearly plane wave conditions, d is a well defined function of v_0. Intuition therefore suggests that under such conditions

$$\left\{\frac{\partial d}{\partial p}\right\}_{v_0 \text{ const.}} = 0 \qquad (28)$$

A little algebra then shows that this is identical with the usual Chapman-Jouguet condition. This form of the condition is beautifully illustrated in figures 1 and 3, which show that the d_3 and d_2 curves touch at the point where there is no change of initial density across the interface.

To complete the discussion it should be further remarked that while for a given v_0, d is insensitive to small changes in p or v or w, it is immediately affected by small changes in q or in atomic composition of the explosive.

Two further examples are given to show what happens when the succeeding explosive is denser than the first. We have used a Kistiakowsky-Wilson equation of state to calculate data for a "plumbatol" taken to be TNT/lead nitrate in the proportions 23.5/76.5 by weight at an initial density of 3.15 gm./c.c. and assuming all the lead to form gaseous Pb O. We obtained $d_2 = 5.68 \cdot 10^5$ cm./sec., $w_2 = 1.39 \cdot 10^5$ cm./sec., $p_2 = 2.49 \cdot 10^{11}$ dynes/sq.cm. We then found that if this explosive were preceded by TNT of initial density 1.6 gm./c.c. (using data calculated by Brinkley and Wilson, loc.cit.) then no carry-over occurred but a weak rarefaction wave reduced the pressure behind the detonation zone by some 10%. On the other hand if it were preceded by PETN at an initial density of 1.727 gm./c.c. then the two values of γ were almost equal but the very high value of q for PETN was more than sufficient to compensate for its low density relative to the plumbatol, so that carry-over occurred. Its effect was to increase the pressure by some 30% but the detonation velocity by under 3%.

Figure 3 illustrates one further point. If the second explosive has a very low density say < .01 gm./c.c. then a very intense rarefaction wave will travel back into the detonation products of the first explosive, accelerating them to a very high velocity. Air shock-wave velocities measured beyond the end of a cylindrical charge detonated from the other end show that this fluid velocity may reach some $8 \cdot 10^5$ cm./sec. Since the second explosive can offer little resistance its detonation products will be driven forward at this speed and the detonation front, travelling at somewhat less than sonic speed relative to those products will have an absolute velocity of some 10^6 cm./sec. i.e. higher than that of the first explosive fired alone. The d_3 curve will therefore have a minimum but details of its shape will depend on the actual equation of state.

7. Conclusions from Part I.

There is little to be added to the discussion of the results already given except to repeat that a constant detonation velocity is no proof of constant conditions behind the detonation front. Again if the initial density of an explosive varies from point to point, by not more than a few percent, the detonation velocity will vary with it and since the relationship between the two is nearly linear, such variations can be tolerated in measurements of detonation velocity as a function of initial density provided the arithmetic mean of that density, along the path of the detonation wave, is known accurately.

Measurements of increased detonation velocities due to carry-over can provide some additional information on the true equation of state of the detonation products. An equation of state is equivalent to a surface in (p, v, T) or (p, v, e) space, and measurements of the variation of the detonation velocity with variations in v_0 and q enable a narrow strip of the latter surface, containing the curve corresponding to normal detonation conditions, to be mapped. Carry-over can be used to widen that strip in the direction of higher values of e but not of p. To extend the strip to higher values of p a study of the head-on impact of two detonation waves might be used.

Part II. The Oblique Case

8. Introduction

We limit ourselves to plane waves and steady conditions. In figure 5 the two detonation fronts OL and OM intersect in the interface OK between the

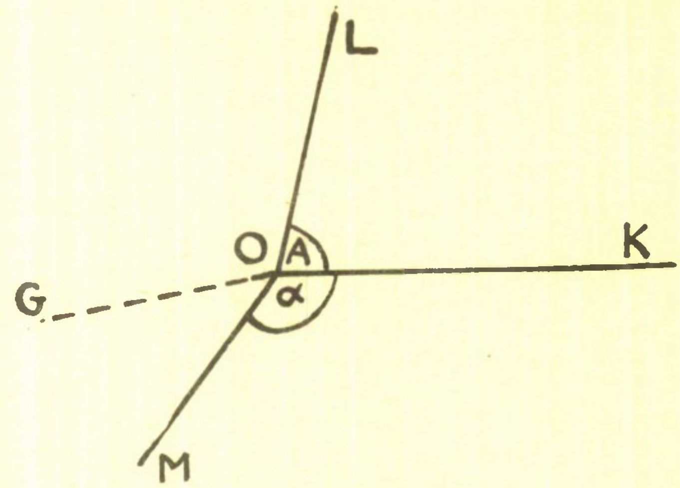


Figure 5.

two explosives. The interface OG between the detonation products will in general not be a continuation of KO. If D and d are the detonation velocities in the upper and lower explosives respectively and A and α the angles KOL, KOM then since O moves along the surface OK we must have

$$D \operatorname{cosec} A = d \operatorname{cosec} \alpha \qquad (29)$$

For any value of A, (29) gives two values of α and conversely. There are four possible cases but we shall only consider the one illustrated namely A acute, α obtuse. The case of A obtuse, α acute can be included by interchanging the properties of the two explosives. The case when both are obtuse is not stable since the point O would in fact travel with a velocity equal to the greater of D and d; the tip of the wedge would therefore be blunted and there would be a region round O, whose dimensions increased linearly with time, inside which conditions would closely resemble the first case with A near $\frac{1}{2}\pi$. The fourth case when both A and α are acute is the intersection of two waves coming from independent sources and so is somewhat artificial since their point of intersection would not in general follow the interface. This case will not be considered here.

It is immediately obvious from (29) that if $D < d$ there will be a range of values of A, symmetrical about $\frac{1}{2}\pi$, for which no steady solution is possible. The same condition governs total reflection in optics but the analogy ceases at that point. For example, let us take a plane wave in the upper explosive bounded on one side by a rigid plane wall parallel to the direction of motion of the wave, so that the latter strikes the interface at its junction with the wall at some definite time (figure 6). A cylindrical wave, of nearly circular cross-section will then start out from the edge of the interface into the lower explosive and will travel along the interface faster than the incident wave. It will therefore drive a new plane wave, with a value of A in the permitted range of obtuse angles, back into the upper explosive. The width of this new plane wave will increase with time so that conditions will not be strictly steady. There will also be a high pressure region due to the intersection of the two waves in the upper explosive.

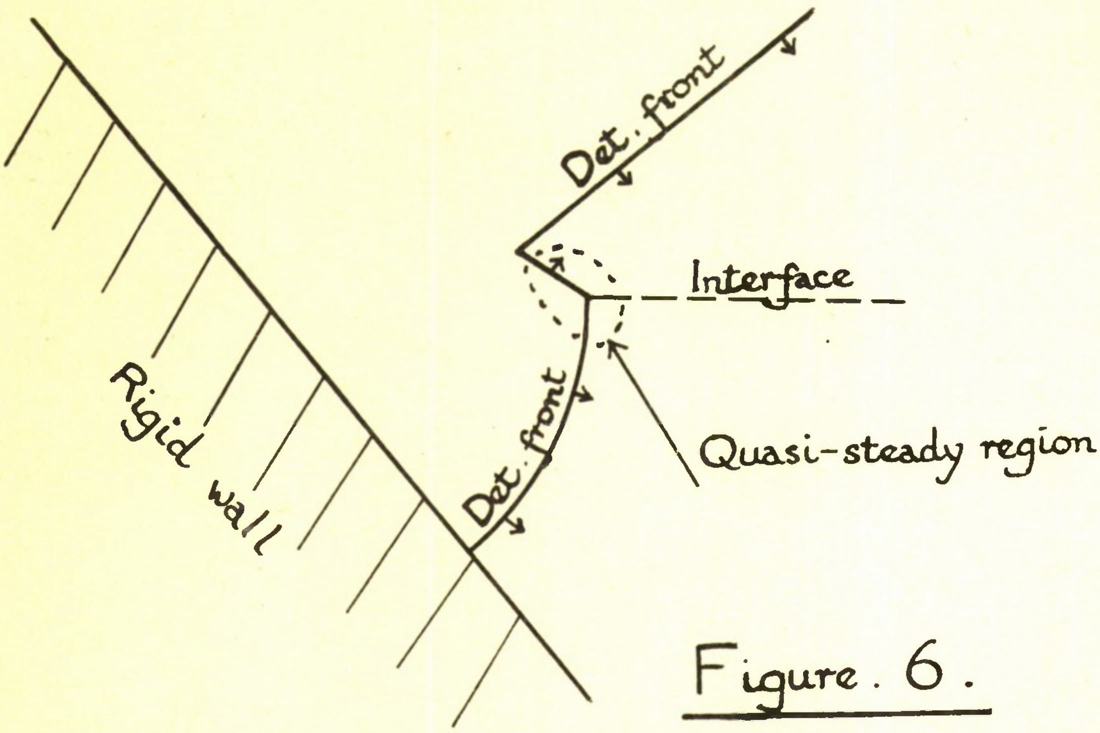


Figure. 6.

A similar change would occur if an initial value of A could be produced, greater than $\frac{1}{2}\pi$ but in the forbidden range. This might be done by placing the lower explosive behind a rigid corner.

If we take a co-ordinate system in which the point O is at rest then the flow is steady and two-dimensional. It is interesting to find, as we shall do, that as $A \to 0$ and $\alpha \to \pi$ our solutions pass over smoothly into those for the one-dimensional unsteady case treated in part I.

Since we ignore the thickness of the reaction zone no dimension is introduced into our problem i.e. conditions are constant along any radius. In the detonation products we may therefore have shock fronts radiating from O, or Prandtl-Meyer expansions centred on O. One condition that must be satisfied is that the pressure is the same on both sides of the interface OG. The fluid velocities on each side must also be parallel to OG but not necessarily equal, so that in general OG will be a slipstream.

If LOM were a shock front, and the lower explosive had a much higher acoustic impedance than the upper, then no solutions could be obtained for values of A exceeding some critical value, and a Mach wave would be formed. The case when the lower explosive is replaced by a perfectly rigid wall has been treated by Polachek and Seeger (1945), who give a formula for the limiting value of A. A plane detonation wave however, travels at sonic speed relative to the detonation products behind it and this condition ensures that the limiting value of A is $\frac{1}{2}\pi$ when the lower explosive is replaced by a rigid wall. It is therefore not surprising to find that a Mach wave is never formed in the case we consider. A Mach wave could of course, be formed if the incident wave were an overdriven one.

The reflection of a detonation wave at a rigid wall is essentially the same as the intersection of two detonation waves inside a single explosive. Hence the above argument shows that Mach waves or jets are never formed under such circumstances i.e. there will be no very high-speed detonation wave produced although there will be a region, between the reflected shocks behind the detonation waves, in which the pressure is increased by a factor of order one to five.

9. Rarefaction waves in the products of detonation

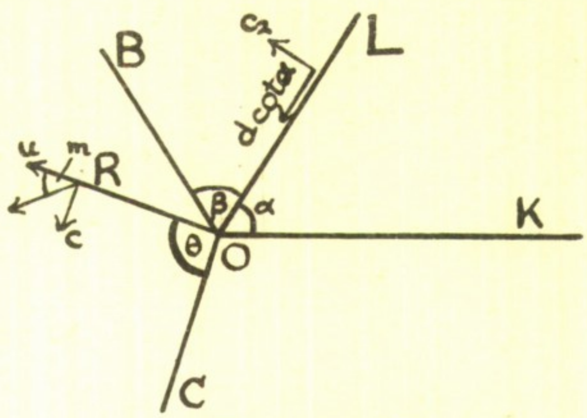


Figure 7.

We assume in the first place that we have only one explosive, the other being replaced by a vacuum (figure 7). We now get a complete expansion, the products travelling parallel to some direction OC with zero pressure and temperature but a very high velocity w_0. We also assume that over the whole range the adiabatic equation for an ideal gas holds, with an exponent γ fitted to conditions in the early stages of the expansion. Since we shall subsequently cut off the expansion at an early stage the use of a wrong value of γ in the later stages is of no importance. For convenience we use small letters for our variables but the results apply equally well to either explosive.

The properties of such a Prandtl-Meyer expansion are well known and expansion behind a detonation front has been treated by Staniukovich (1947). We shall use his results with a few minor corrections. The fluid velocity w at any point R of the wave has a tangential component equal to the local velocity of sound c and if we denote the radial component by u then we have from Bernoulli's theorem

$$w_0^2 = u^2 + (\gamma + 1)\, c^2/(\gamma - 1) \tag{30}$$

$$= d^2\{\cot^2\alpha + \gamma^2/(\gamma^2 - 1)\} \tag{31}$$

If θ be the angle COR then since $du/d\theta = c$ we can deduce

$$\left.\begin{aligned} u &= w_0 \cos n\,\theta \\ c &= n\, w_0 \sin n\,\theta \\ \text{where} \quad n^2 &= (\gamma - 1)/(\gamma + 1) \end{aligned}\right\} \tag{32}$$

The local Mach angle m of the flow, which equals the angle between the flow direction and the radius vector is given by

$$\tan m = n \tan n\,\theta \tag{33}$$

If α is acute there will be a direction OB where COB is the angle θ_0 given by

$$n \tan n\,\theta_0 = c_2/d \ \cot\alpha$$

$$= \frac{\gamma}{\gamma + 1} \tan\alpha \tag{34}$$

and at any point on OB the flow will have tangential and radial components c_1 and d cot α while between OB and OL the flow will be uniform. Since the tangential and radial components of flow at any point on OL are c_1 and $-$ d cot α respectively we find by trigonometry that if β is the angle LOB then

$$\cot \tfrac{1}{2}\beta = \frac{\gamma}{\gamma + 1} \tan \alpha \tag{35}$$

On the other hand if α is obtuse then the radial component of flow along OL is positive and OB coincides with OL, i.e. expansion starts immediately from the back of the detonation zone.

The angle through which the flow is turned from its direction KO before encountering the detonation is

$$\epsilon = \alpha + \beta + \theta_0 - \theta + m - \pi \tag{36}$$

measured in a counter-clockwise direction for the upper explosive and a clockwise direction for the lower explosive.

At the ends of the two expansions, if they occur simultaneously, the sum of the two values of ϵ must therefore be zero. Since the expansion is not taken very far the flow is never radial; the expansion therefore stops on a line making an angle m with the interface while between that line and the interface the flow is uniform.

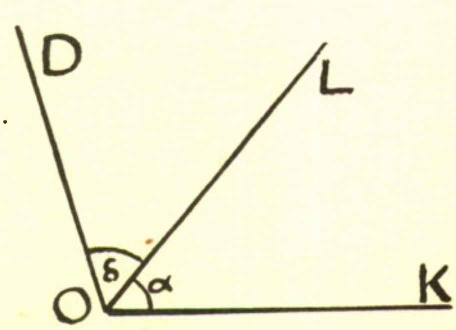
The relation between pressure p_3 and angle ϵ can be derived from (32) and the adiabatic relation

$$(p_3/p_2) = (c_3/c_2)^{2\gamma/(\gamma-1)} \tag{37}$$

10. Shock waves in the products of detonation

Since a shock wave travels at supersonic speed relative to the fluid it is approaching, it will catch up the detonation wave if α is obtuse and give an increased detonation velocity. In figure 8 let OD be the shock front, making an angle δ with the detonation front OL. Then conditions will be

Figure 8.



uniform between OL and OD and again between OD and the interface. Let p_2, u_2, w_2 be pressure, radial velocity and tangential velocity of the fluid approaching OD and p_3, u_2, w_3 those of the fluid leaving OD then since OD is stationary we have

$$(\gamma + 1)\, w_3 = (\gamma - 1)\, w_2 + 2\, c_2^2/w_2$$

and

$$p_3 = p_2 + \rho_2\, w_2(w_2 - w_3) \tag{38}$$

where

$$w_2 = o_2(\cos\delta + \frac{\gamma + 1}{\gamma} \cot\alpha \sin\delta)$$

These equations determine the shock strength in terms of the angle δ.

For a given shock strength, i.e. a given value of w_2, (38) gives in general two solutions for δ. For α acute, the component of velocity along OL is negative and we can easily see that the shock wave with the smaller value of δ is unstable. This is because a reduction in δ, produced by a displacement of OD towards OL would cause a reduction in the component, normal to OD, of the fluid flow between OD and OL, so that the shock must continue to move forward into that fluid. An increase in δ would produce the opposite effect. For α obtuse the converse is true but we cannot have the shock wave ahead of the detonation. Hence only one case can arise, namely α acute and δ the larger root of (38).

The detonation and shock fronts produce changes in fluid velocity of $d - c_2$ and $w_2 - w_3$ normal to their respective fronts. Hence the flow beyond OD has components $\{d \cot \alpha \cos \alpha + c_2 \sin \alpha - (w_2 - w_3) \sin (\alpha + \delta)\}$, $\{(d - c_2) \cos \alpha + (w_2 - w_3) \cos (\alpha + \delta)\}$ parallel to KO and to the downward normal to KO. If ϵ is the angle through which the flow direction has been turned, measured in the anti-clockwise direction, then $\tan \epsilon$ will equal the second component divided by the first. We have now expressed both p_3 and ϵ as functions of the parameter δ and so can construct a curve giving ϵ as a function of p_3. This curve will join on to the corresponding curve for the rarefaction wave given by (36) and (37) of the previous section, where p_3 and ϵ are expressed in terms of θ as parameter. It should be noted that, at the join, $p_3 = p_2$ but $\epsilon \neq 0$ since the flow is deflected by the detonation wave through an angle

$$\epsilon_0 = \tan^{-1} \{(\sin \alpha \cos \alpha)/(\gamma + \cos^2\alpha)\} \tag{39}$$

11. Enhanced detonation velocity

We now consider the case when the shock has caught up the detonation front. Cases of practical importance are those for α obtuse. The acute case would appear to be possible e.g. if the first explosive were gaseous, provided a suitable method of initiation could be found, but we shall see later that this is not so.

Using w_3 for the tangential velocity i.e. the component of velocity behind the detonation front and normal to it then p_3 is given in terms of w_3 by equation (15). Adding on the radial component $d_3 \cot \alpha$ we find for the deflection of the flow

$$\left.\begin{aligned} &\epsilon = \alpha - \tan^{-1}(w_3/d_3 \cot \alpha) \text{ for } \alpha < \tfrac{1}{2}\pi \\ \text{or} \quad &\epsilon = \alpha - \pi - \tan^{-1}(w_3/d_3 \cot \alpha) \text{ for } \alpha > \tfrac{1}{2}\pi \end{aligned}\right\} \tag{40}$$

We must remember that d_3 is no longer the natural detonation velocity but is given by

$$d_3 = p_3 v_0/w_3 \tag{15}$$

which may be derived immediately from (1) and (2). If the velocity of the point O is to be kept constant then α must be varied with p_3 in the appropriate manner.

12. The nature of the flow

From figure 9 it will be seen that the (ϵ, p_3) curve for the first explosive consists of a loop TUVW with tail ST while for the second explosive it consists of the curve stuv terminating at the point v on the p_3 axis. In general, these curves intersect in more than one point and it is our business to decide which point of intersection gives the conditions which will actually obtain.

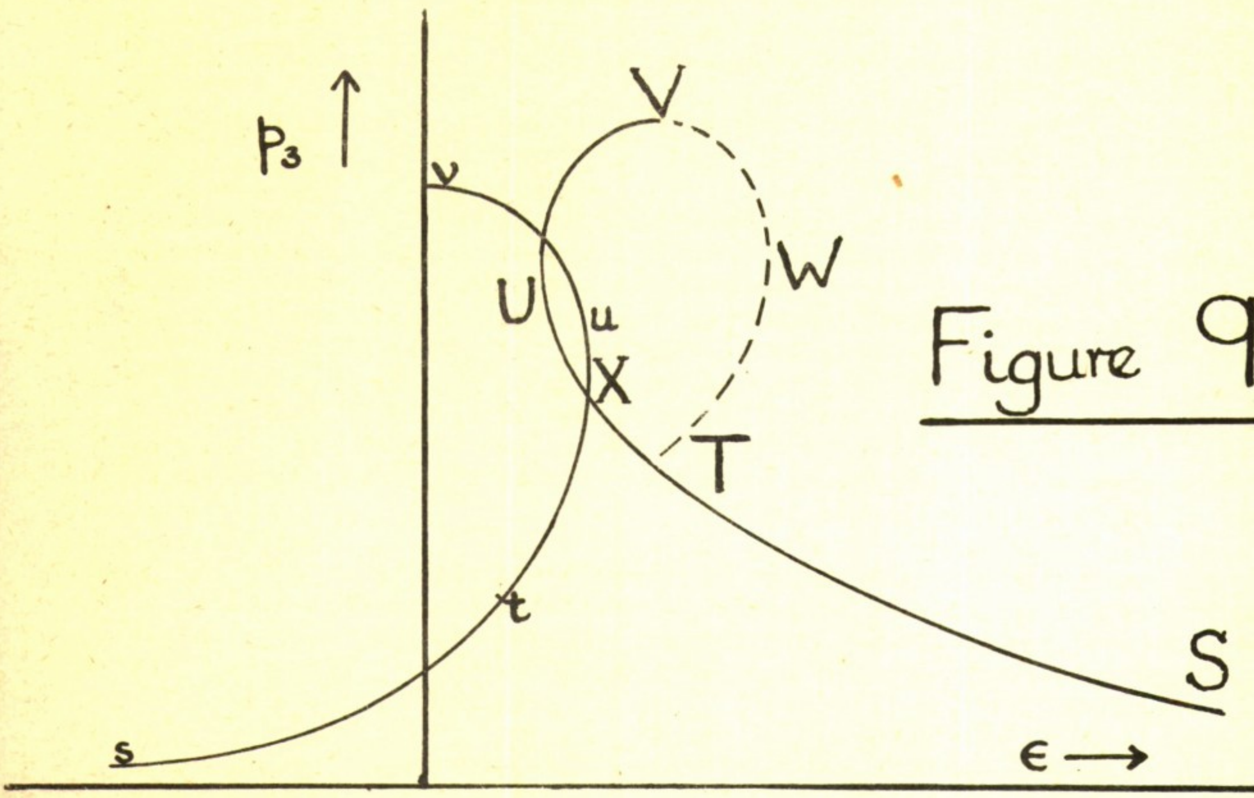


Figure 9.

The right-hand half VWT of the loop TUVW is given by the smaller of the two solutions of (38) for δ. As explained in § 10, this solution gives an unstable shock wave in the first explosive and so is ruled out. Hence the half loop VWT gives no acceptable solutions.

From the shapes of the two curves we see that we can get at most two intersections on the valid part of the curve STUV. Let one of these intersections be at the point X and let N and n be the slopes $d\epsilon/dP_3$, $d\epsilon/dp_3$ of the first and second curves respectively at X. Suppose $n > N$ then if the pressure be increased the deviation of flow ϵ for the first explosive is increased less than that of the second explosive, so that the gases tend to separate and a rarefaction is produced which lowers the pressure; conversely if the pressure is reduced a pressure wave is set up which will restore it. The condition is therefore stable. The same argument shows that a point for which $n < N$ represents an unstable condition. It is clear from figure 9 that one condition obtains at one point and the other at the other; we therefore get at most one stable solution, and this is the one with the lowest pressure. For the case of a detonation wave passing from PETN to plumbatol treated in the next section it is gratifying to find that the stable solution passes over smoothly to the head-on solution as the angle $A \rightarrow 0$.

Since the curve for the first explosive does not cross the $\epsilon = 0$ axis we can have cases where the two curves do not intersect at all, giving no stable solution. This happens when equation (29) cannot be satisfied and we get a quasi-steady solution as described in § 8. When the two curves touch we must have the critical case, in which $\alpha = \frac{1}{2}\pi$.

In figure 9 we can mark on each curve the point T or t corresponding to normal detonation conditions. If the point of intersection lies above or below T there will be a shock or rarefaction wave behind the detonation front in the first explosive. If it lies below t there will be a rarefaction behind the detonation front in the second explosive but if it lies above t there will be an increased detonation pressure and velocity i.e. a carry-over effect.

The point T is at the end of the loop for the first explosive (the curve terminates at this point). We could draw such a loop for the second explosive by drawing the curve for the angle $\alpha' = \pi - \alpha$, which would be the mirror image of stuv in the p_3 axis. If the first explosive were very weak the curve STUV might intersect the tail of this image curve. We should then have a solution with both A and α acute. The intersection would probably be above T so that there would be a shock following the detonation front in the upper explosive but no carry-over effect. The criterion given earlier in this section shows that all such solutions are unstable.

13. Numerical results

In figures 10 and 11 p_3 is plotted as a function of ϵ for the two explosives PETN and plumbatol, of which details were given in section 6. PETN has been taken for the upper explosive with angle of incidence (A in figure 5) equal to 60° and 80° respectively. α has been taken obtuse, of course, for the lower explosive. The loop for the first explosive increases rapidly in size as α is reduced. Only the left-hand side of this loop applies to our case since the larger value of ϵ, for a given p_3 is derived from the smaller root for δ i.e. the unstable case. The sign of ϵ has been reversed for the second explosive so that the point of intersection gives conditions at the interface.

The points of intersection of the curves have been calculated for other values of A and the results are given in the following table. Since $p_2 = 24.9 \cdot 10^{10}$ dynes/sq.cm. for plumbatol we see that carry-over ceases at an angle A a little greater than 60°.

A	p_3 dynes/sq.cm.	ϵ
90°	16.5×10^{10}	4° 30′
80°	19.5 ,,	6° 10′
60°	26.5 ,,	7° 45′
40°	30.7 ,,	7° 2′
20°	31.4 ,,	4° 10′
0	32.5 ,,	0

Figure 12 has been drawn for a detonation wave passing from PETN at high density (1.727 gm./c.c.) to PETN at a very low density (0.241 gm./c.c.) using the same data as for part I and taking A = 60°. In this case we get two solutions, of which the one with lower pressure is stable. For this combination we see that carry-over will persist until A is increased to considerably more than 60°.

14. Conclusion

There is little to add to the discussion given in § 12 except to consider the effect of the finite reaction-zone thickness. Equalisation of pressure across that part of the interface which separates the two reaction zones will cause that part of the front to be curved, just as the front of a detonation wave travelling along a cylindrical stick of explosive is curved near the outer edge (see Eyring et.al., loc.cit.). The introduction of a unit of length implies that our picture, in which fluid properties depend on angle alone, is no longer correct, but since the finite thickness of the reaction zone can only produce a finite change in momentum or energy of the system per unit area of interface, the true solution must soon approach that given here as we get away from the interface. Our solution is therefore an asymptotic one, valid everywhere except near the interface, and so will give the correct behaviour of the major portion of the wave.

References

Brinkley, R. and Wilson, E.B., 1943, O.S.R.D. 1707.

Chambre, P. and Lin, C., 1946, Jour. Aero. Sci. 13, 537.

Eyring, H., Powell, R.E., Duffey, G.H. and Parlin, R.B., 1949, Chem. Rev. 45, 69.

Hicks, B.L., Montgomery, D.J. and Wasserman, R.H., 1947, Jour. Appl. Phys. 18, 891.

Jones, H., 1950, Proc. Roy. Soc. (in press)

Paterson, S., 1948, Proc. Phys. Soc. 61, 119.

Staniukovich, K.P., 1947, C.R. (Doklady) Acad. Sci. U.R.S.S. 55, 311.

Taylor, G.I. and Maccoll, J.W., 1935, W.F. Durand's Aerodynamic Theory, Vol. III, Division H. (Springer, Berlin)

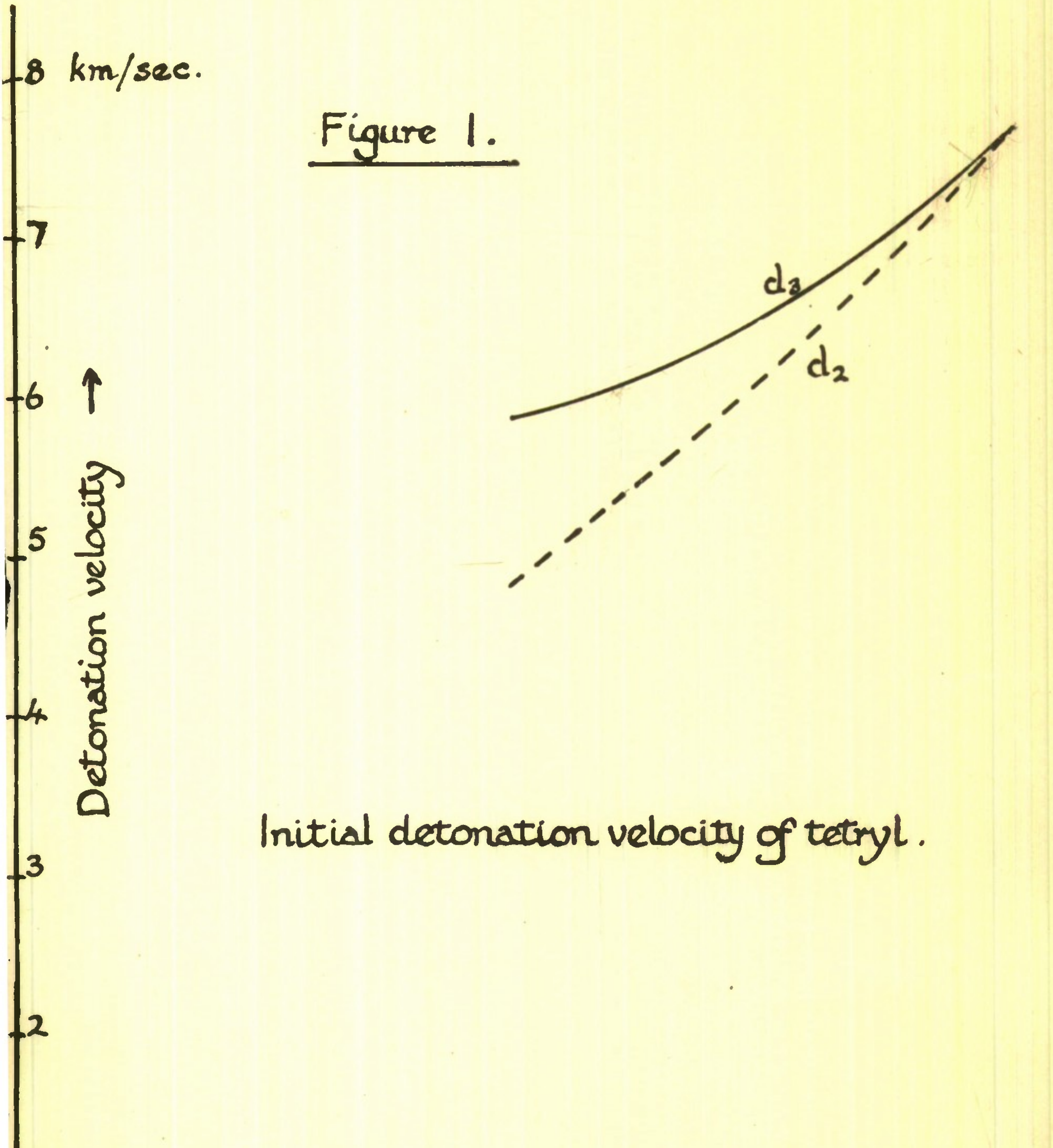
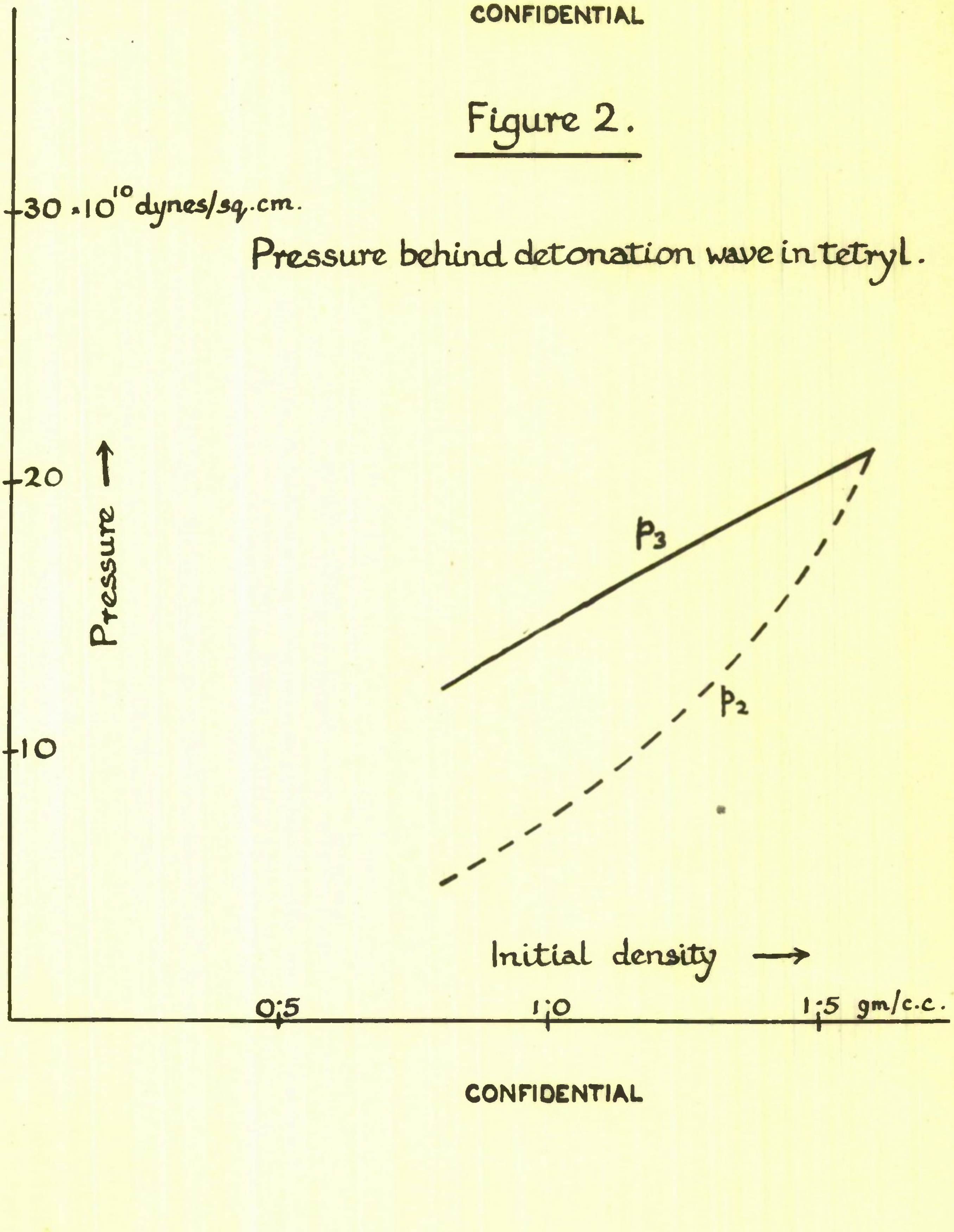


Figure 1.

Initial detonation velocity of tetryl.

Figure 2.



30 × 10^{10} dynes/sq.cm.
Pressure behind detonation wave in tetryl.
20
Pressure
10
p_3
p_2
Initial density
0·5
1·0
1·5 gm/c.c.

Figure 3.

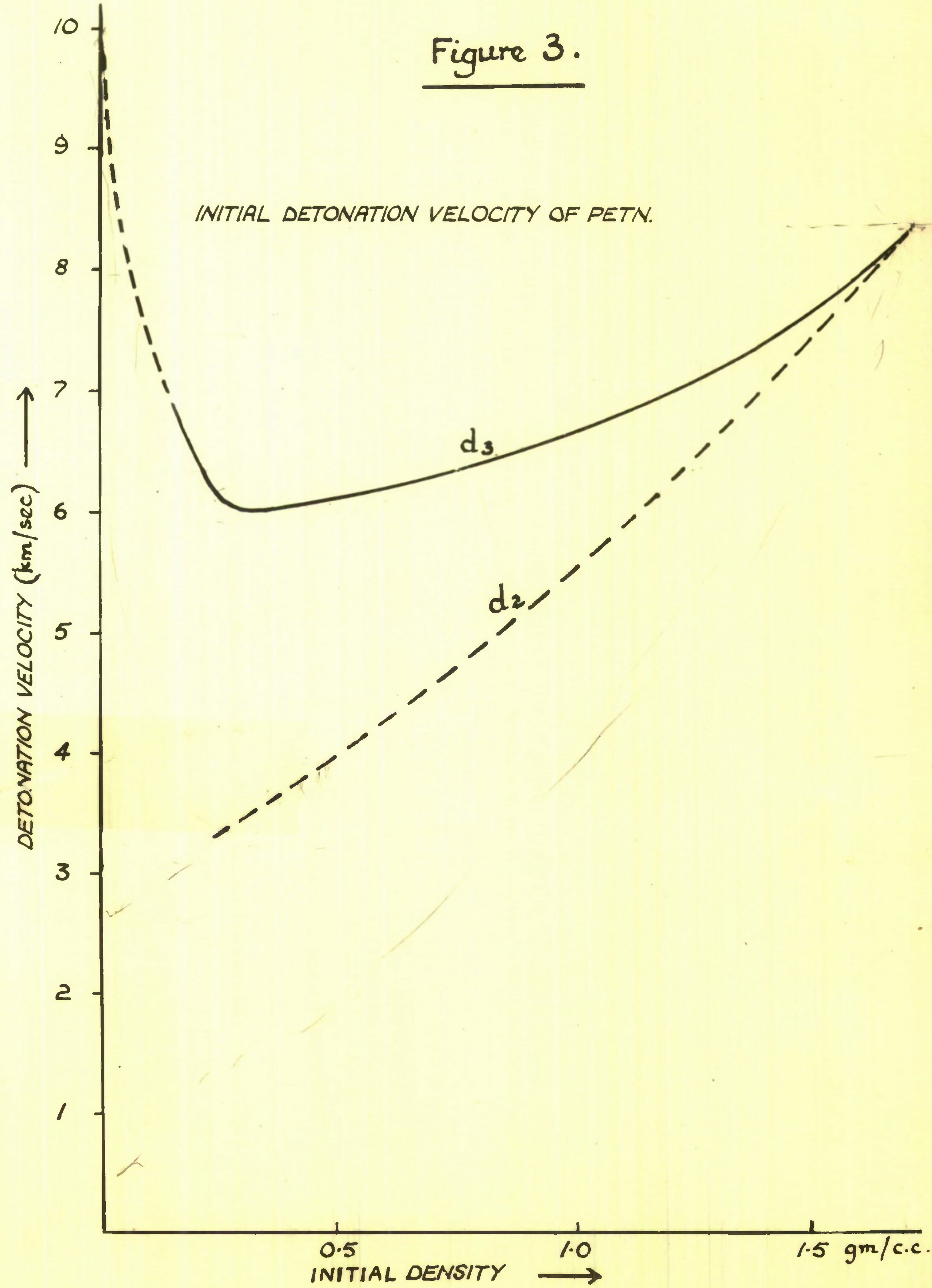
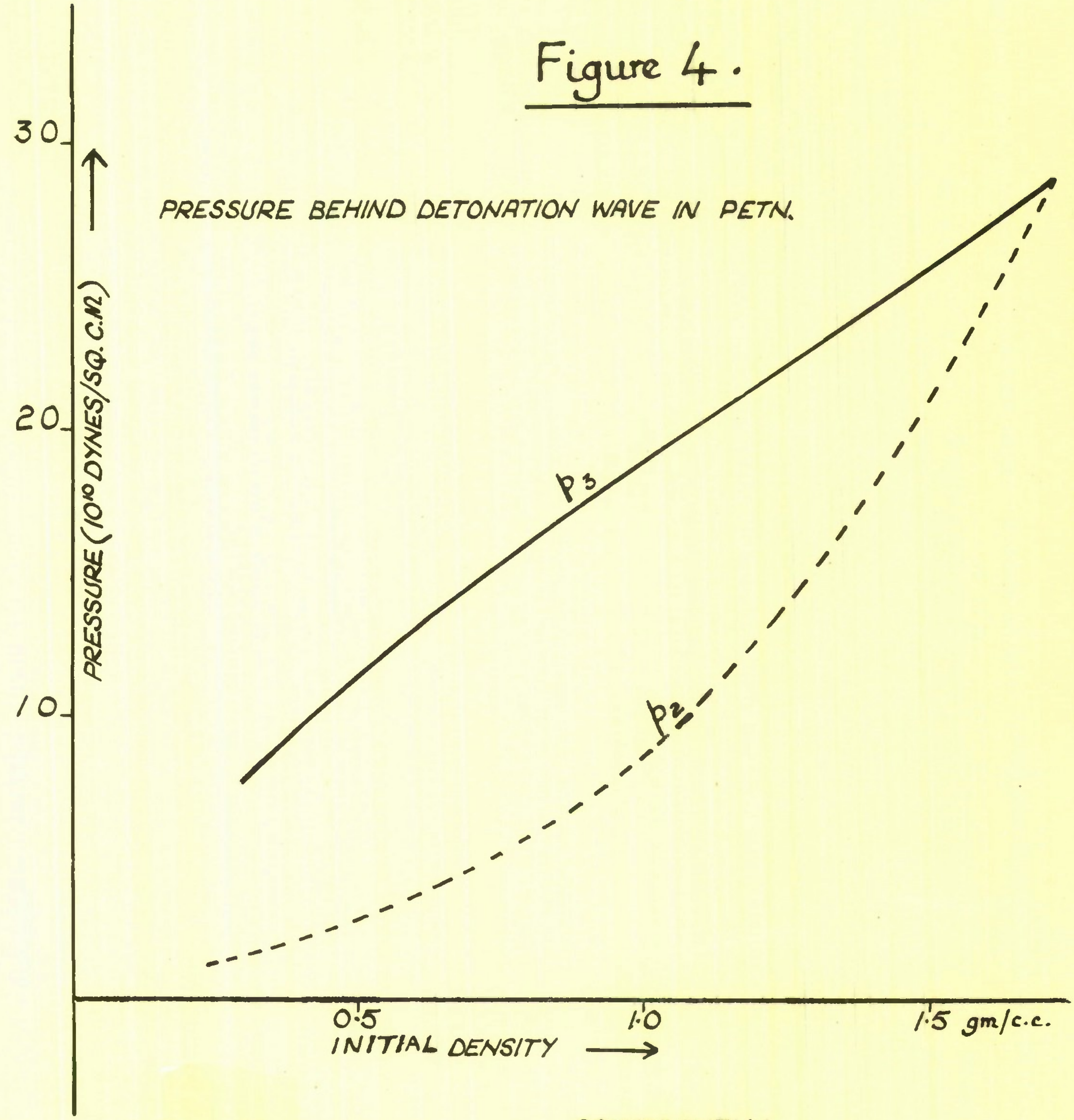

INITIAL DETONATION VELOCITY OF PETN.
DETONATION VELOCITY (km/sec)
10
9
8
7
6
5
4
3
2
1
d_3
d_2
0·5
1·0
1·5 gm/c.c.
INITIAL DENSITY

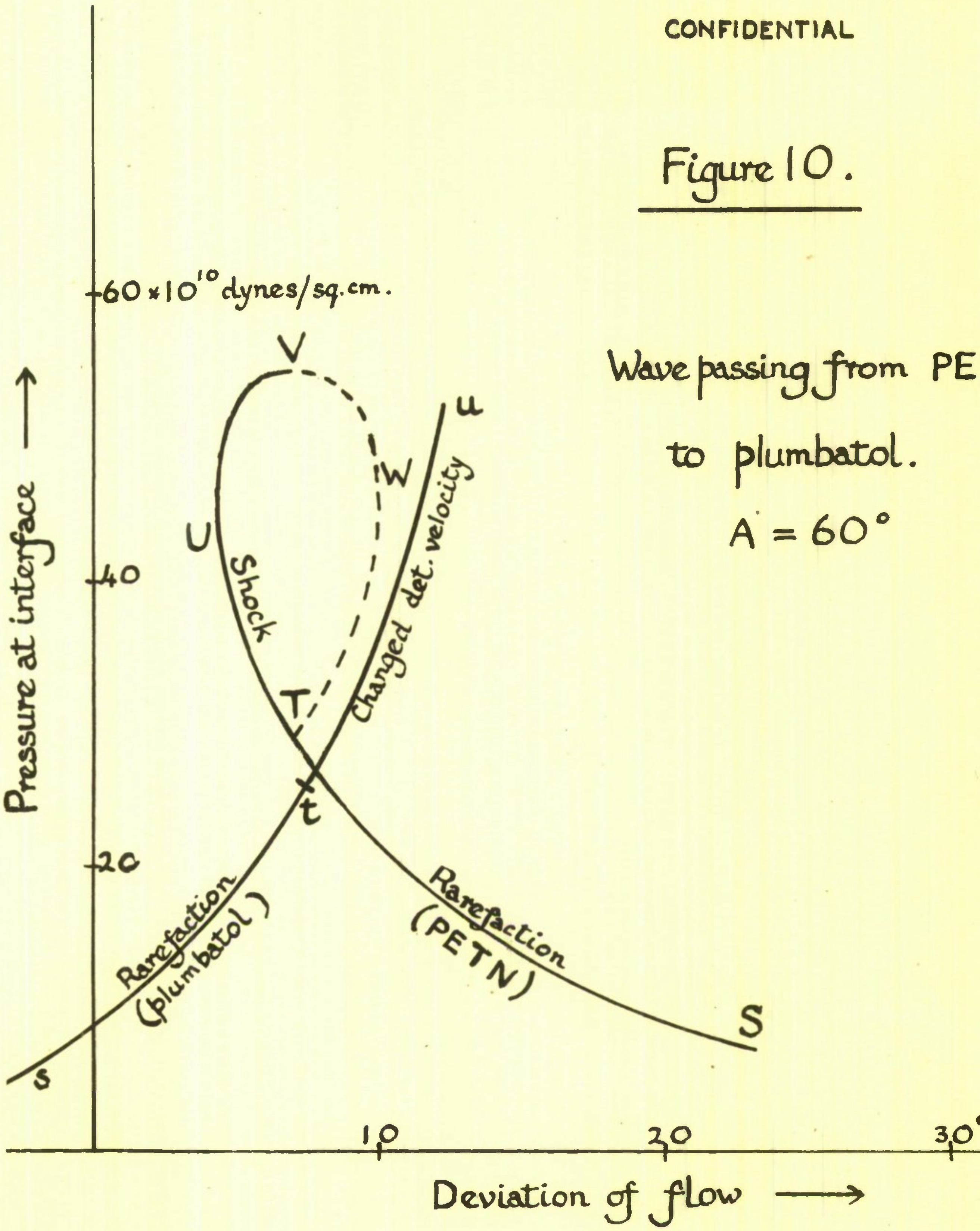
Figure 4.



30
20
10
PRESSURE (10¹⁰ DYNES/SQ.C.M.)
PRESSURE BEHIND DETONATION WAVE IN PETN.
p3
p2
0·5
1·0
1·5 gm/c.c.
INITIAL DENSITY

CONFIDENTIAL

Figure 10.



Wave passing from PETN to plumbatol.

$A = 60°$

CONFIDENTIAL.

~~CONFIDENTIAL~~

Figure 11.

Wave passing from PETN to plumbatol.

A = 80°

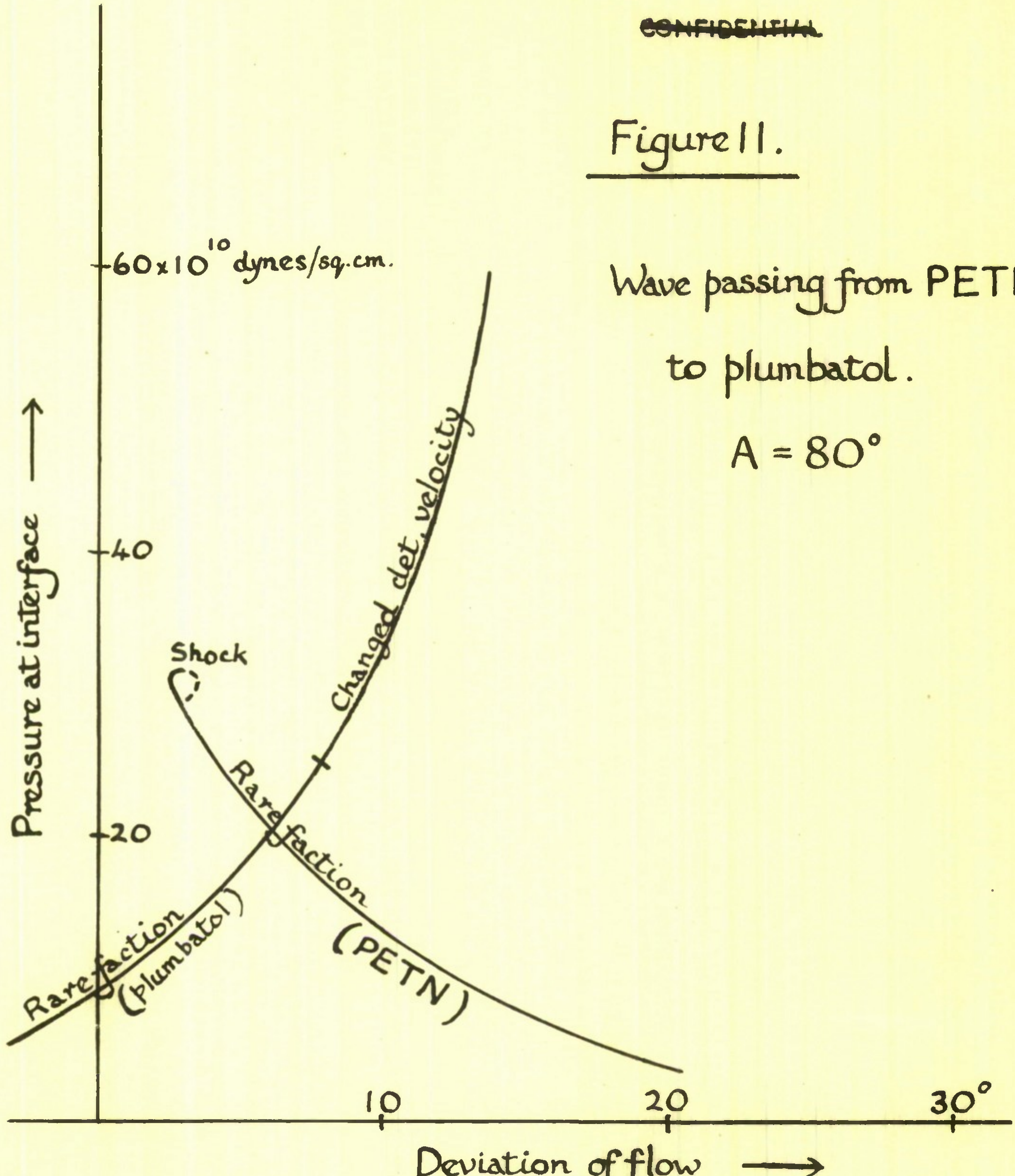


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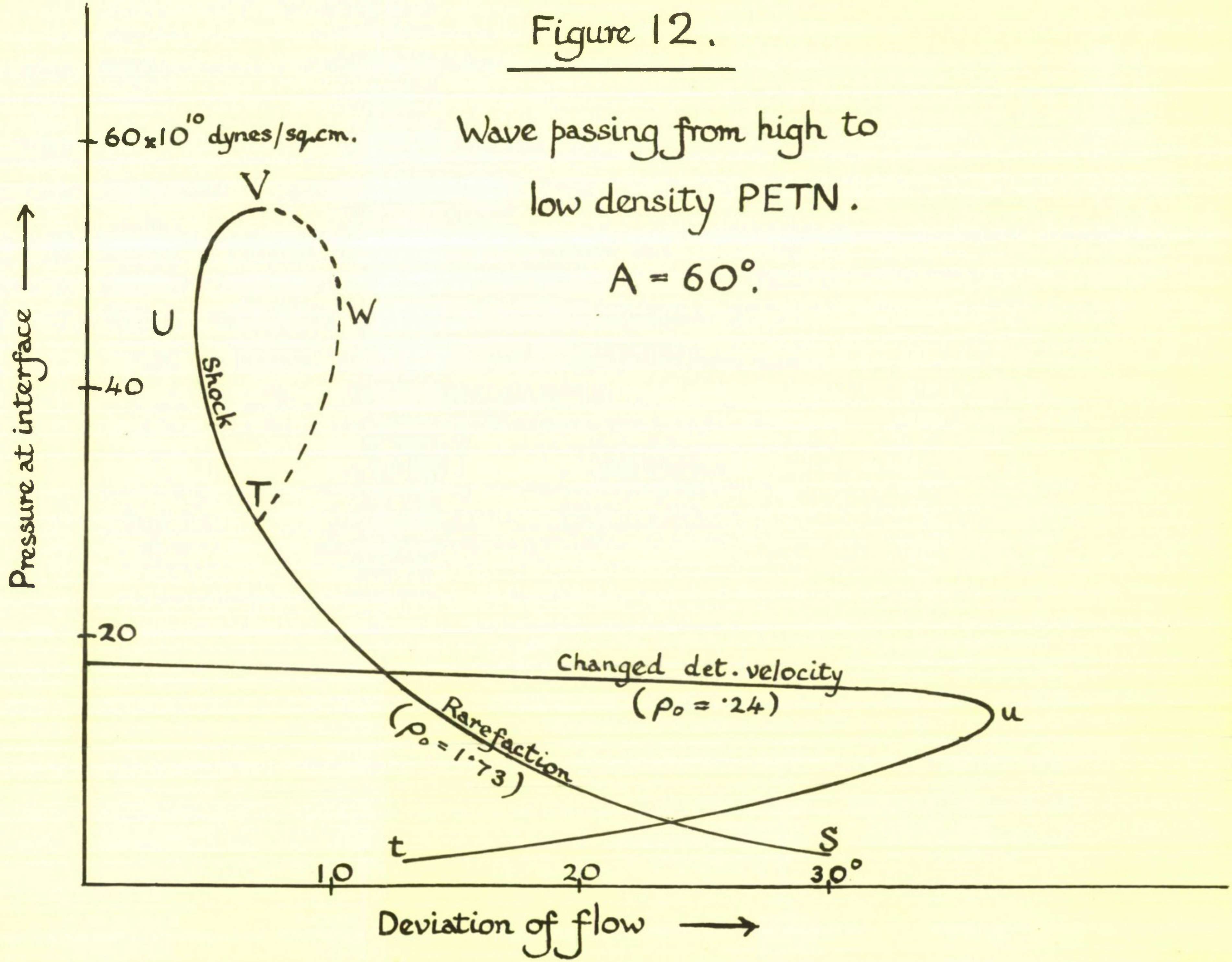
Figure 12.

Wave passing from high to low density PETN.

$A = 60°$.



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Covering dates 1950
Availability Open Document, Open Description, Normal Closure before FOI Act: 30 years
Former reference (Department) Report No 22/50
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